The Hong Kong University of Science and Technology

Department of Mathematics

MPhil THESIS EXAMINATION

On the univalence of some Schwarz Maps

By

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ABSTRACT

The study of Univalent function has a long history and is amongst the most fundamental questions in complex function theory. Nehari studies oscillation of second order ODE and Schwarzian derivatives. His research is initialed by the equation $d^2y/dz^2 + p(z)y = 0$. In 1949, he gave a proof of necessary criteria and sufficient criteria of univalence, $|p(z)| \leq 6/(1-z^2)^2$ and $|p(z)| \leq 2/(1-z^2)^2$ respectively. In the same year, Hille gave an example showing that it is necessary and sufficient that $p(z)(1-z^2)^2$ is at the interior or on the boundary of the cardioid $A = -2e^{i\phi} - e^{2i\phi}, -\pi < \phi \leq \pi$. Hille' result concides with Nehar's only on the real axis of A-plane. Hille's example has complex monodromy. We follow the study of univalent Schwarz map of hypergeometric function defined on unit disk with complex parameters. Two criteria for univalence and non-univalence are derived. Before study of hypergeometric function, we strengthen Hille's example that his Schwarz map gives one-to-one, finitely many-to-one, and infinitely-to-one mapping depending on the value of A relative to the cardioid curve introduced above.

Date: **11 August 2021, Wednesday**

Time: **2:30 p.m.**

Venue: **Online via Zoom**
https://hkust.zoom.us/j/95124861990 (Passcode: Schwarz)

Thesis Examination Committee

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Thesis Supervisor	**:**	**Prof. Yik Man CHIANG, MATH/HKUST**
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(Open to all faculty and students)

The student's thesis is now being displayed on the reception counter in the General Administration Office (Room 3461).